Definition of moment of inertia

Objects of the experiment

- Measuring the period of oscillation of a thin transverse rod with weights on a torsion axle as a function of the distance of the weights from the torsion axle.
- Confirming the proportionality between the moment of inertia of the weights and the square of the distance.
- Determining the restoring torque of the torsion axle.

Principles

The moment of inertia is a measure of the inertia that a body exhibits when a torque acts on it causing a change of its rotational motion. It corresponds to the inertial mass in the case of translational motions. In rotational oscillations, for example, the period of oscillation T is the greater, the greater the moment of inertia J of the oscillating system is. More specifically:

$$T = 2\pi\sqrt{\frac{J}{D}} \qquad \text{(I)}$$

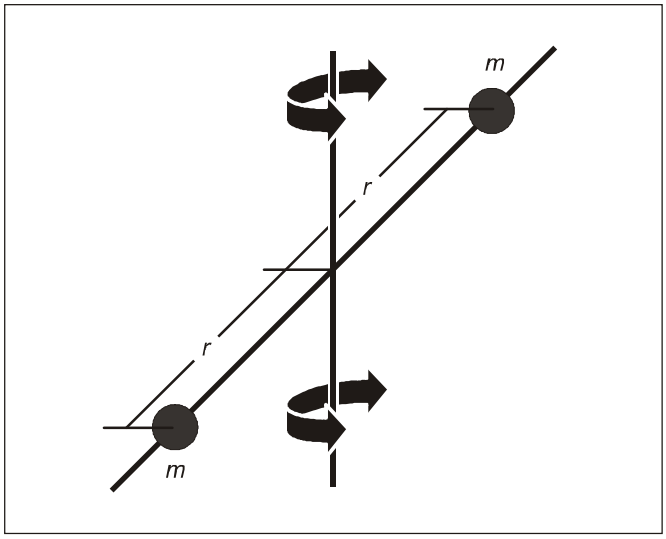
D: restoring torque

The moment of inertia of a pointlike mass m moving on a circular path with radius r is

$$J_1 = m \cdot r^2 \qquad \text{(II)}.$$

The moment of inertia of two equal masses m that are rigidly connected and have the same distance r from the axis of rotation is

$$J_2 = 2 \cdot m \cdot r^2 \qquad \text{(III)}.$$



In both cases, the moment of inertia is proportional to the square of the distance r.

In the experiment, the rigid connection between the two masses is established by means of a thin rod whose middle is fixed to the torsion axle (see Fig. 1). After deflection from the equilibrium position, the system oscillates with the period of oscillation T. From Eq. (I) it follows that

$$J = D \cdot \left(\frac{T}{2\pi}\right)^2 \qquad \text{(IV)}.$$

However, the moment of inertia is composed of the moment of inertia J_2 of the two weights and the moment of inertia J_0 of the rod:

$$J = 2 \cdot m \cdot r^2 + J_0 \qquad \text{(V)}.$$

Therefore the period of oscillation T_0 of the rod without weights is measured in another measurement, which leads to

$$D \cdot \left(\frac{T}{2\pi}\right)^2 = 2 \cdot m \cdot r^2 + D \cdot \left(\frac{T_0}{2\pi}\right)^2$$

or

$$T^2 = \frac{8 \cdot m \cdot \pi^2}{D} \cdot r^2 + T_0^2 \qquad \text{(VI)}.$$

Thus a linear relation between the square of the period of oscillation T and the square of the distance r is obtained. From the slope of the straight line,

$$a = \frac{8 \cdot m \cdot \pi^2}{D} \qquad \text{(VII)},$$

the restoring torque D can be calculated if the mass m is known.

Fig. 1 Schematic illustration

Apparatus

1 torsion axle	347 80
1 stand base, V-shape, 20 cm	300 02
1 stopclock I, 30 s / 15 min	313 07

Setup and carrying out the experiment

The experimental setup is illustrated in Fig. 2.

– Fix the middle of the transverse rod to the torsion axle and arrange the weights symmetrically at a distance of 30 cm from the torsion axle.

– Mark the equilibrium position on the table.

– Rotate the transverse rod to the right by 180° and release it.

– Start the time measurement as soon as the transverse rod passes through the equilibrium position and stop the measurement after five oscillations.

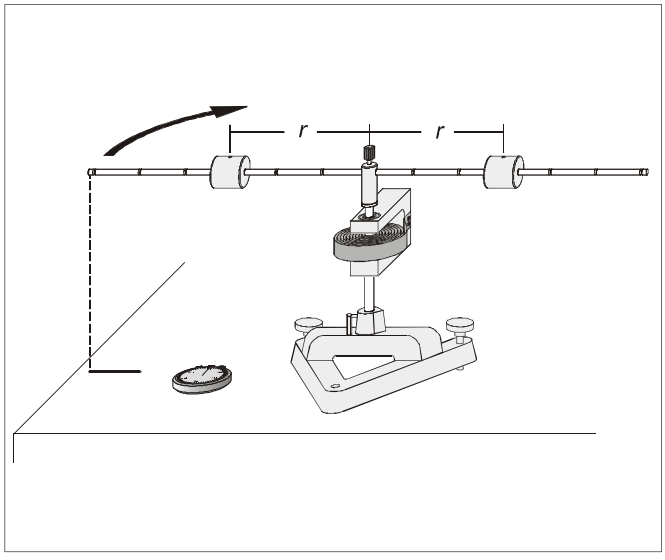
– Repeat the measurement four times, alternately deflecting the rod to the left and to the right.

– Calculate the period of oscillation T from the mean value of the five measured values.

– One after another reduce the distance to 25 cm, 20 cm, 15 cm, 10 cm and 5 cm, each time repeating the measurement.

– Remove the weights, and repeat the measurement.

Fig. 2 Experimental setup for determining the period of oscillation.



Measuring example

Individual weight to be put on the rod: m = 0.24 kg

Table 1: measured duration of five oscillations for various distances r between the torsion axle and the weights and mean value of the period of oscillation T calculated from the measured values

$\frac{r}{\text{cm}}$	$\frac{5 \cdot T}{\text{s}}$					$\frac{T}{\text{s}}$
30	45.2	45.1	45.0	44.9	45.2	9.016
25	38.0	38.1	38.0	38.2	38.0	7.612
20	31.5	31.6	31.6	31.7	31.3	6.308
15	25.5	25.4	25.2	25.6	25.4	5.084
10	19.6	19.5	19.8	19.7	19.7	3.932
5	15.0	15.5	15.3	15.7	15.4	3.076
Without weights	13.1	13.3	13.3	13.4	13.4	2.660

Evaluation

From Table 1 we obtain T_0 = 2.660 s and therefore $T_0^2 = 7{,}076\ \text{s}^2$. The squares of the oscillation periods of the system with weights are listed in Table 2.

Table 2: list of the values of r^2 and T^2

$\frac{r}{\text{cm}}$	$\frac{r^2}{\text{cm}^2}$	$\frac{T^2}{\text{s}^2}$
30	900	81.288
25	625	57.943
20	400	39.791
15	225	25.847
10	100	15.461
5	25	9.462

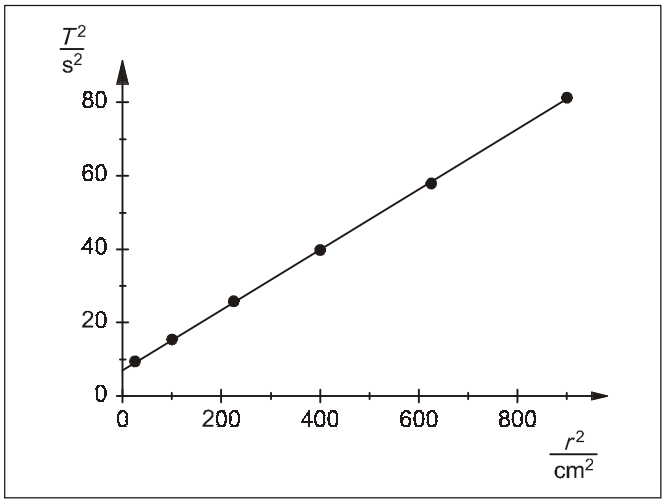


Fig. 3 Square of the period of oscillation T^2 as a function of the square of the distance r^2

In Fig. 3, the relation between T^2 and r^2 is shown graphically. Within the accuracy of measurement, the values drawn in the figure lie on a straight line with the slope

$$a = 821 \frac{s^2}{m^2}.$$

As the mass m is known (m = 0. 240 kg), the restoring torque D can be calculated from the slope according to (VII).

We obtain D = 0.023 Nm.

Result

The period of oscillation of a transverse rod with weights on a torsion axle is the greater, the greater the distance of the weights from the axis of rotation is because the moment of inertia of the system increases.

The moment of inertia of a “pointlike” mass with respect to an axis of rotation is proportional to the square of the distance from the axis of rotation.

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